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HOLDUP IN TWO PHASE FLOW

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Abstract—A wide range of experimental holdup data have been analysed on the basis of the general correlations of Chen & Spedding (1983). For upward inclined flow, holdup data in the range $(\bar{R}_G/\bar{R}_L) = 4.0$ to 275 were handled using a modification of the Chen & Spedding method, and for the case of $(\bar{R}_G/\bar{R}_L) \leq 4.0$, the modified Armand equation was found to be suitable. Horizontal stratified flow was examined using the Bernoulli equation, and shown to be a limiting case of the free draining of a tube initially filled with liquid. For downward inclined stratified flow, the Manning equation predicted the holdup accurately for low liquid rates and small angles of inclination. In addition, for these two cases of horizontal and downward stratified flow, the holdup also was examined in terms of the critical depth of flow as determined using the total energy relation.

INTRODUCTION

Spedding & Chen (1979a, b) have suggested a general correlation for holdup prediction in horizontal two phase flow. The correlation consisted of a plot of holdup ratio $\bar{R}_G/\bar{R}_L$ against the flow rate ratio Q_G/Q_L. The form of the correlation was suggested by Butterworth (1975) and subsequently has been justified theoretically by Chen & Spedding (1983). A limited range of horizontal experimental data were used by these latter authors to show that holdup data may be broadly classified into three major groups, depending on the type of flow pattern, with different relationships being found to represent the data for each group. For bubble and slug type flows, the holdup was given by the equation due to Armand (1946),

$$\frac{\bar{R}_G}{\bar{R}_L} = \frac{1}{0.2 + 1.2/(Q_G/Q_L)} \quad [1]$$

which was shown to be a special case of the theoretical development due to Nguyen & Spedding (1977). With stratified type flow the holdup was given by a series of relations which were derived using a simple separated flow model. Annular flow, on the other hand, was satisfactorily represented by a semi-empirical correlation for the $\bar{R}_G/\bar{R}_L$ values above 4.

It is the purpose of this work to review and extend the application of this type of correlation technique to the horizontal two-phase flow situation and to endeavour to expand its applicability to the case of inclined flow.

Horizontal flow

The holdup data of Spedding & Nguyen (1976) and Chen & Spedding (1979) obtained for air–water flow in a 4.54 cm i.d. pipe over a 2 m length are plotted in figures 1 and 2. In figure 1 the stratified flow data were excluded from the plot while in figure 2 the stratified type flow data were included. The flow regimes given on the figures unfortunately are not discernable following reproduction. This is particularly the case in figure 2 where the data points for the various stratified regimes virtually are obliterated by the curves which in all cases lie directly over the points. The accuracy of the $\bar{R}_G$ data was better than $\pm 1\%$. It should be noted that use of the factor $\bar{R}_G/\bar{R}_L$ magnifies the scatter of the results in general while an order of magnitude increase in $\bar{R}_G/\bar{R}_L$ approximately halves the expected scatter of data points.

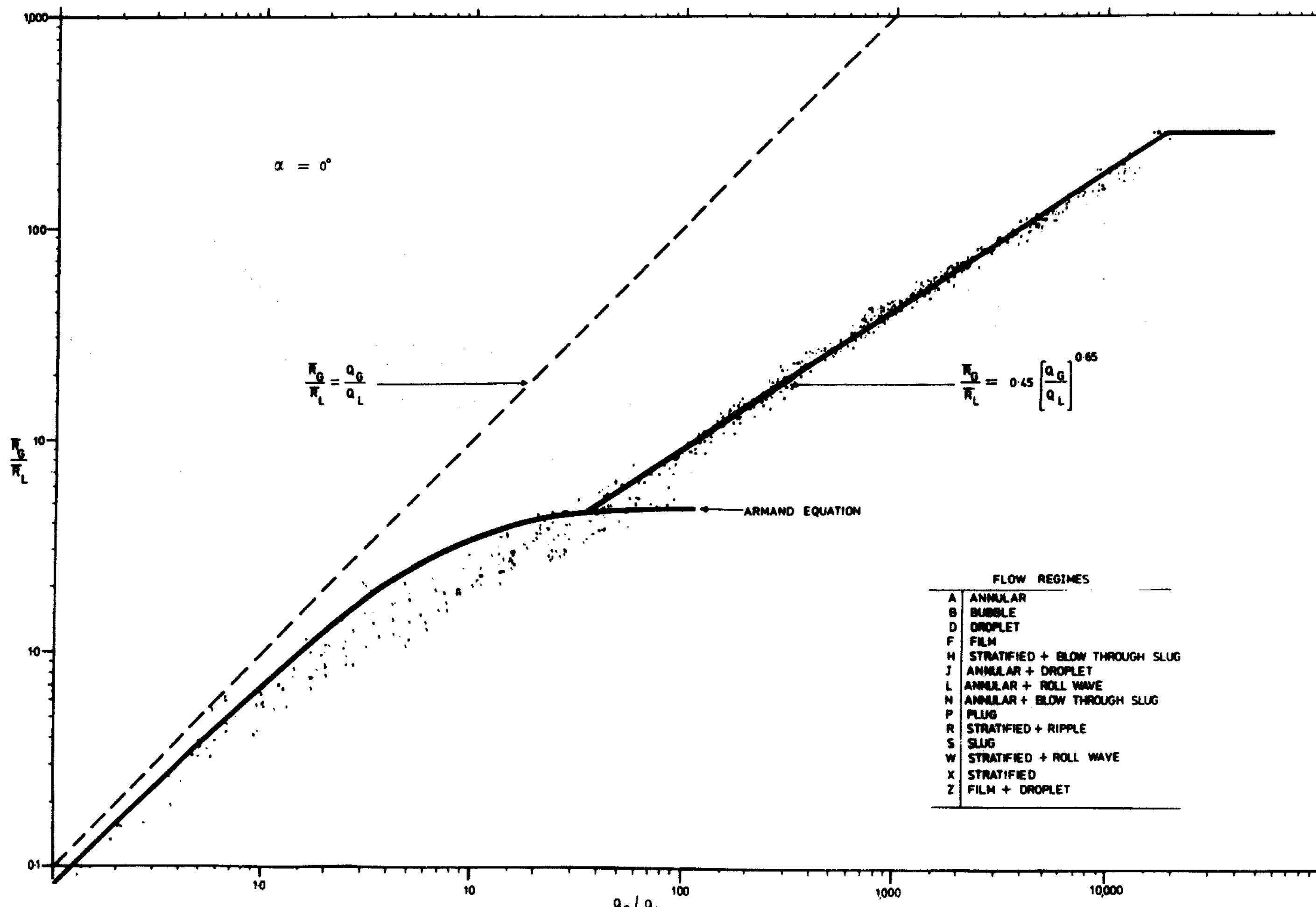

Figure 1. Horizontal two-phase air–water flow holdup data for a 4.54 cm i.d. pipe. Stratified flow data are excluded in order to simplify the presentation.

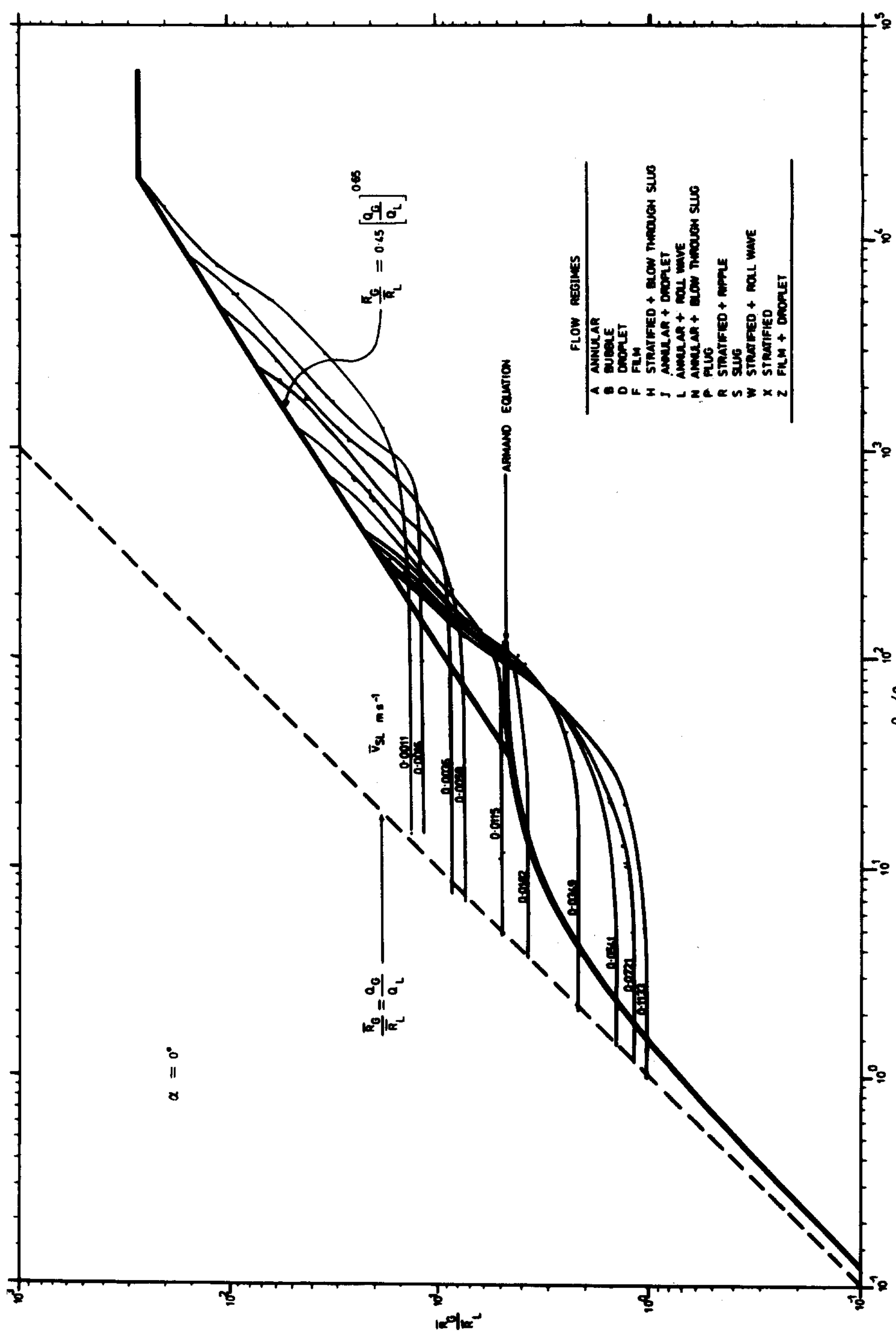

Figure 2. Horizontal holdup data for two-phase air–water flow for a 4.54 cm i.d. pipe. The flow is in the stratified regime only. $\bar{V}_{SL}$ = superficial liquid velocity.

The correlation of figure 1 shows that the annular type flow data fitted the semi-empirical correlation suggested by Spedding & Chen (1979a, b).

$$\bar{R}_G/\bar{R}_L = 0.45\,[Q_G/Q_L]^{0.65} \quad [2]$$

for values of $\bar{R}_G/\bar{R}_L \geqslant 4$, where $\bar{R}_G$ and $\bar{R}_L$ are the gas and liquid holdups and Q_G and Q_L the gas and liquid volumetric flow rates. Below $\bar{R}_G/\bar{R}_L = 4$ the data are predominantly of the slug flow type and do not give a good fit to the Armand equation. In addition a wide scatter is in evidence. The bubbly flow regime data, on the other hand, give close agreement with the Armand relation. The basic reason for the wide scatter in the slug flow regime data and the lack of fit to the Armand equation, is that the tube length over which the holdup was measured was too short to enclose the entire length of either a slug or a bubble with certain types of long slug flow patterns. Thus under conditions where long liquid slugs were present the measured liquid holdup would be biased towards the high side whereas the bubbly flow regime would be correctly measured in the apparatus since there is little sensitivity to the effect of tube length with the bubble regime. Therefore the holdup data of Spedding & Nguyen (1976) and Chen & Spedding (1979) can be expected to be inaccurate for the slug type flow regimes, giving value for $\bar{R}_L$ which are on the high side.

At values of $Q_G/Q_L \geqslant 15{,}000$ the data in figures 1 and 2 gave a constant value of $\bar{R}_G/\bar{R}_L$. In this region of high gas flow and low liquid flow, the liquid is held in two forms; as droplets which are swept along with the gas core as a homogeneous type mixture and as a liquid film on the inner wall of the pipe. Armand (1946) showed that the liquid film reduces with increasing gas flow rate to a constant asymptotic value of wall flow, i.e. to a constant liquid film thickness on the inside wall of the pipe, independent of input liquid flow above the critical value. For the experimental conditions of Nguyen & Spedding (1976) the liquid holdup $\bar{R}_L = 0.00364$ gave a liquid film thickness of 0.083 mm at $Q_G/Q_L \geqslant 15{,}000$. The liquid film in this region of flow was observed to be continuous round the inside wall of the pipe. However, when the gas rate was lifted above $Q_G/Q_L = 60{,}000$ the liquid film was broken and commenced to strip off the inner wall of the tube. In such a region of flow rate the $\bar{R}_G/\bar{R}_L$ value was observed to climb steeply and presumably eventually coincided with the homogeneous line where $\bar{R}_G/\bar{R}_L = Q_G/Q_L$. The maximum gas rate obtainable for the experiments of Spedding & Nguyen (1976) and Chen & Spedding (1979) was not sufficient to give this region quantitatively on figures 1 and 2, but did allow the qualitative nature of the regime to be observed.

A detailed comparison was made in figures 3 and 4 between the form of correlation suggested in figure 1 and a wide range of data for horizontal flow. Figure 3 compares the calculated annular steam–water data of Harrison (1975) for 20 cm internal diameter pipe over the pressure range of 0.45–$1.23 \times 10^6\,\mathrm{kg\,m^{-1}\,s^{-2}}$. The data demonstrates reasonable agreement with the correlation suggested in this work, but are of more importance in that they show that the correlation is applicable to the steam water system in large diameter pipes.

In figure 4 the data from a number of sources are plotted and again exhibit a general agreemnent with the correlation suggested in this work. Some detail needs to be presented about these data in order to highlight the breadth of application of the suggested correlations. The steam water data of Isbin *et al.* (1957, 1958) and of Fujie (1964) for pressures between 7×10^5 and $7 \times 10^6\,\mathrm{kg\,m^{-1}\,s^{-2}}$ in 1.23 cm i.d. pipe gave good agreement with the suggested correlation. Incidently the same is true of the data collected by Von Glahn (1962) from various sources which are not included in figure 4 in the interests of clarity. The following air–water data gave good agreement with the correlation; Chrisholm & Laird (1958) for a 2.69 cm i.d. pipe, the Dartmouth correlation of Wallis (1969) and Farmer *et al.* (1978) for a 2.5 cm i.d. pipe. The annular flow data of Hewitt *et al.* (1961)

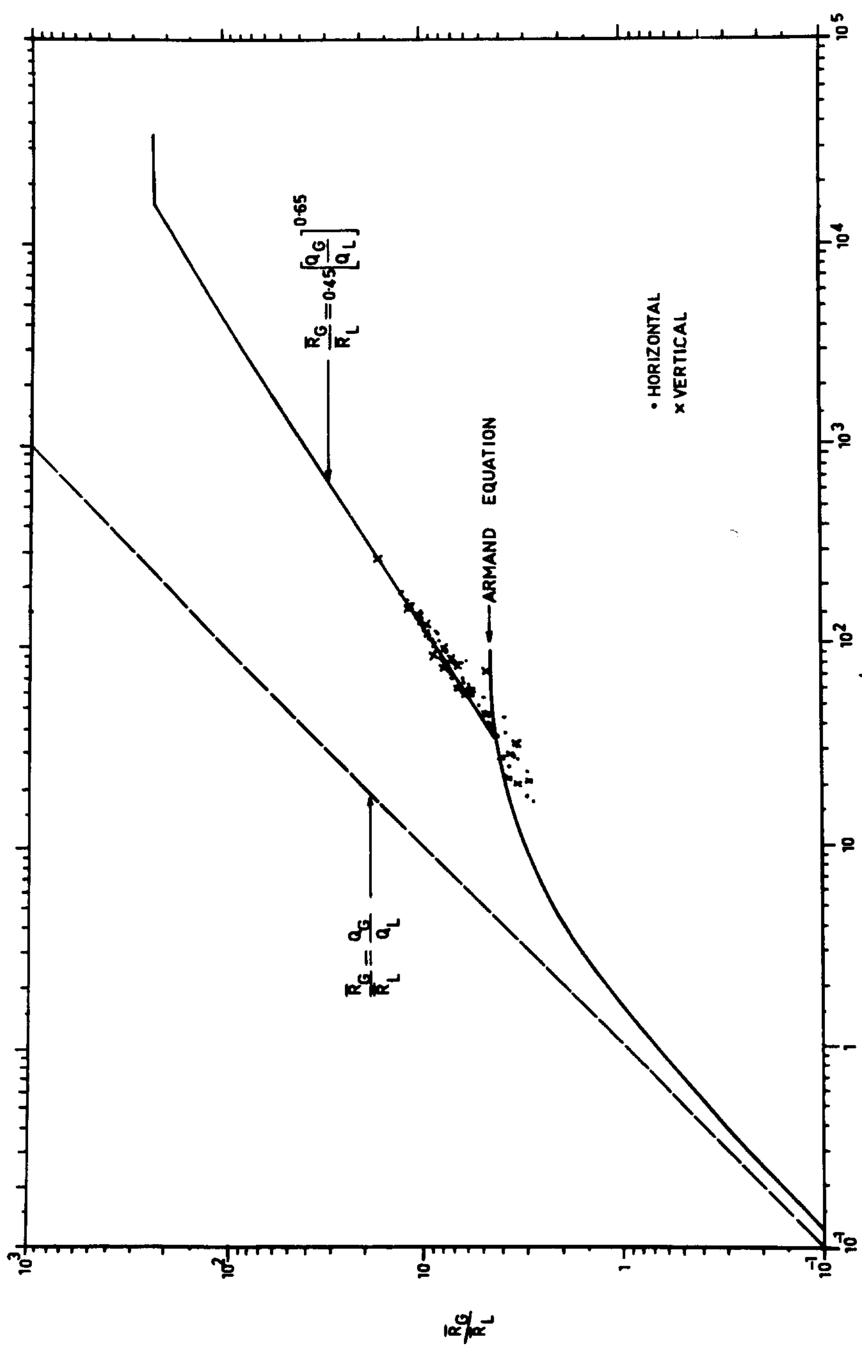

Figure 3. Comparison between the steam–water holdup data of Harrison (1975), for horizontal and vertical flow and the correlations suggested in this work for horizontal two phase flow.

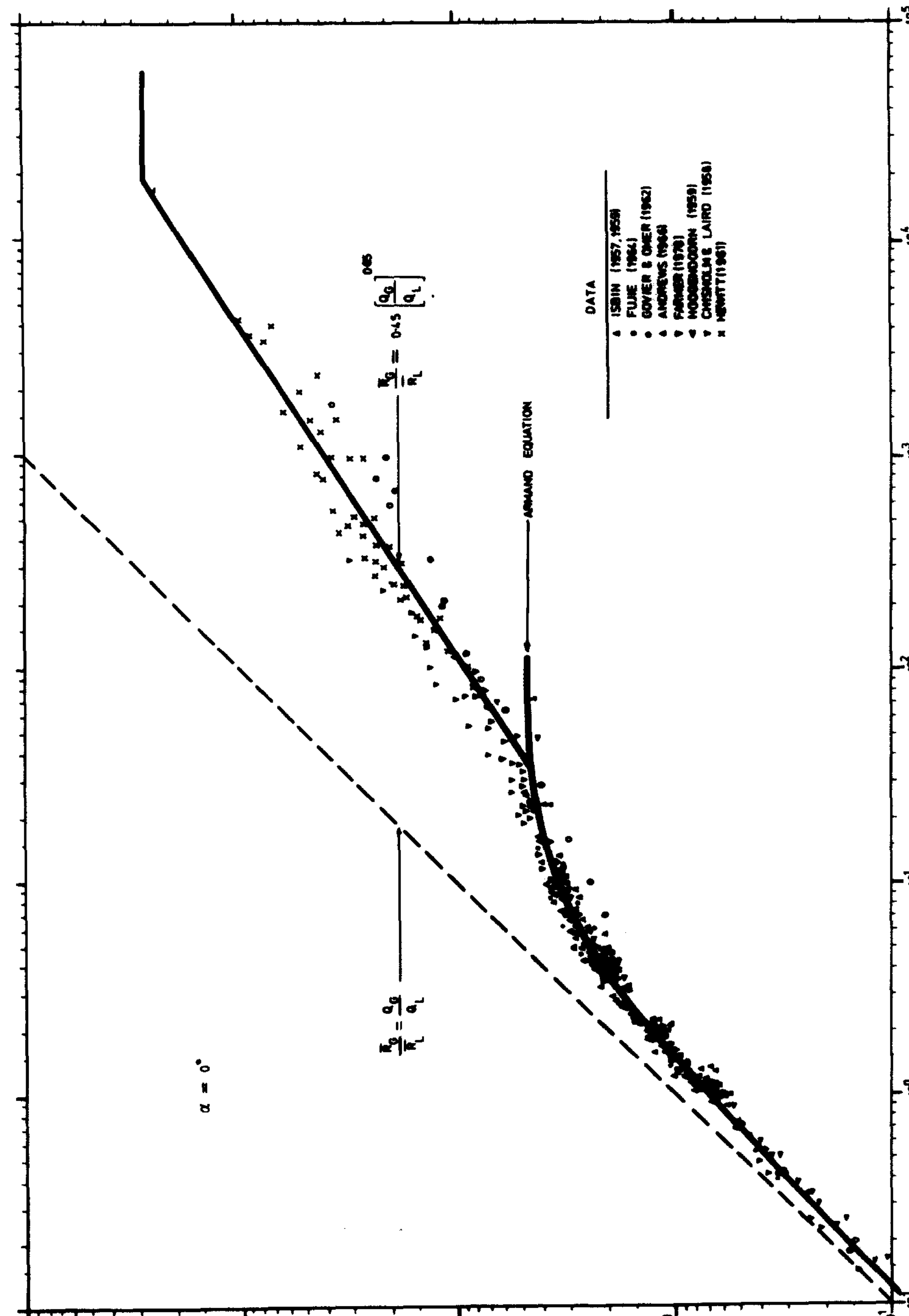

Figure 4. Comparison between various holdup data and the correlations suggested in this work for horizontal two phase flow.

for a 3.18 cm i.d. pipe showed reasonable agreement with the correlation. The data of Andrews (1966) obtained on 515 m of 5.25 cm i.d. pipe for water-natural gas flow gave a good agreement with the Armand relation. Later work by Eaton (1970) on the same apparatus and system but with a 10.5 cm i.d. line again gave good agreement with the Armand relation for slug type flows. The rest of the data obtained by these two workers was in the stratified regions and while they were not applicable to the current discussion it is worth noting that general agreement was obtained with the correlations in figure 2. The data of Hoogendoorn (1957) for oil–water flow in a 14 cm i.d. pipe gave good agreement with the Armand relation for slug type flows but other data were ignored here as they were in the stratified regimes, but they gave reasonable agreement with figure 2. The data used by Lockhart & Martinelli (1949) proved to be very scattered and were not used in this check. About a third of the data of Johnson & Abou-Sabe (1952) did not give agreement with the correlation but this was to be expected since they were collected under conditions pertaining to a study of heat transfer.

For horizontal two phase flow with the stratified type flow regimes the data in figure 2 initially exhibit a series of horizontal lines at lower gas rates which depend on superficial liquid velocity $\bar{V}_{SL}$ but eventually at higher gas rates join into the holdup correlation of Spedding & Chan (1979a, b) which has already been obtained for annular type flow. Data from other literature sources such as Beggs (1972) have not been obtained in a systematic manner, i.e. by setting the liquid rate and altering the gas rate, so give individual points which require interpolation. However, it appears much of the available data do give reasonable agreement with figure 2. Spedding & Nguyen (1978) suggested that this type of flow regime has parallels with open channel flow. This aspect is examined in detail in the following section using the techniques outlined by Chow (1959) and Henderson (1966).

Applying the Bernoulli equation to the case of horizontal stratified flow for a circular conduit as given in figure 5.

$$\frac{P}{\rho_L g} + \frac{V_L^2}{2g} = y_1 + \frac{\bar{V}_{L1}^2}{2g} = \text{etc} = E \quad [3]$$

where P is the hydrostatic pressure, ρ_L is the liquid density, y is the liquid depth, g is the gravitational acceleration, $\bar{V}_L$ is the average liquid velocity over a channel cross section and E is the specific energy. For the case of inviscid flow the specific energy must be constant, so

$$(E - y)A_L^2 = Q_L^2/2g \quad [4]$$

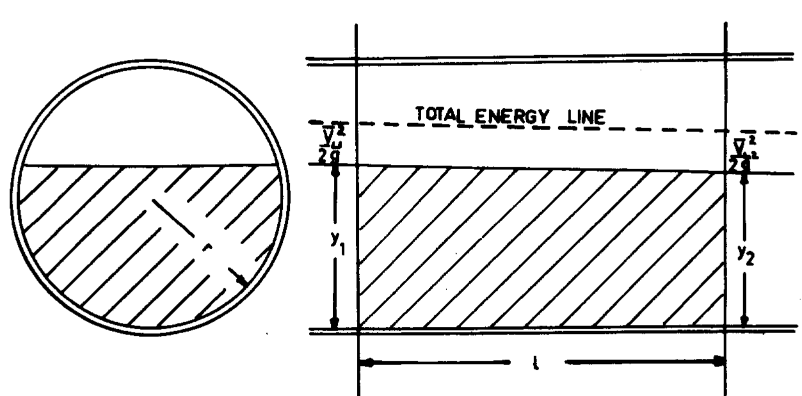

Figure 5. Schematic representation of free surface channel flow in a horizontal circular conduit.

where A_L is the cross-sectional area of liquid flow in the conduit which must be a function of y, the liquid depth. Since for steady state conditions, the volumetric liquid flow rate must be a constant, then [4] is a cubic with two real roots which are asymptotic to two equations as shown in figure 6. The relations between A_L and y used in the calculation of figure 6 are given in figure 7. It is observed that by using the method of presentation given in figure 7 a straight line relation with A_L can be obtained over a wide range. At a given flow rate Q_L there is a minimum specific energy E_c which occurs at the critical depth y_c where the Froude number ($\mathrm{Fr} = \bar{V}_L^2/gy_c$) is equal to one. Further at this point of critical

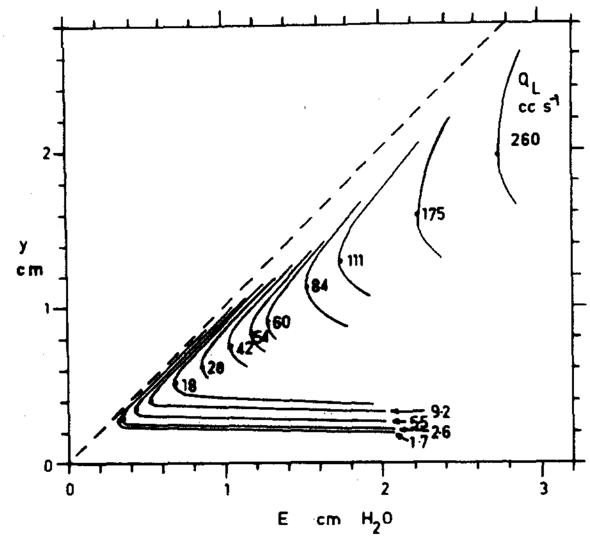

Figure 6. Calculated liquid depth against specific energy for free surface channel flow in a horizontal circular conduit. Q_L = liquid volume flow rate.

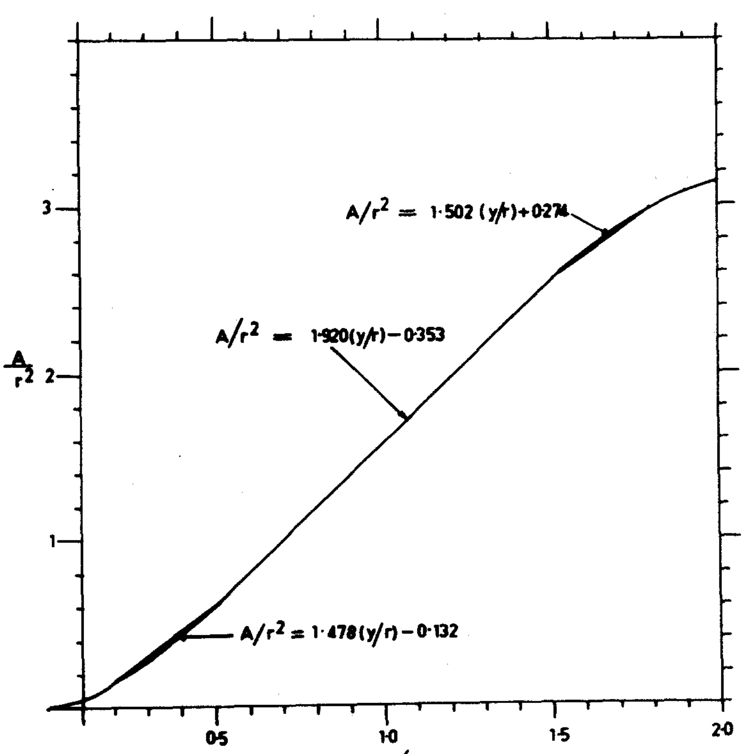

Figure 7. The relationships between the area of liquid flow in a circular pipe and liquid depth.

flow the conduit will give the maximum flow rate for a given specific energy. The appendix details the method of calculation for the data of figure 6.

In figure 8 the critical depth of flow is presented for the various experimental conditions employed by Spedding & Nguyen (1976) and Chen & Spedding (1979) when collecting the data used in obtaining figure 2. The holdup data are converted to y the depth of flow in the pipe, by calculating out the area of liquid flow,

$$A_L = A_T\bar{R}_L \qquad [5]$$

where A_T is the total cross sectional area of the pipe. The depth is found by use of figure 7. It will be observed that the actual experimental liquid depth obtained is much greater than the critical depth calculated from [4]. In fact the actual depth corresponds to twice the critical depth in the low liquid flow range. However, as the liquid flow rate exceeds about $Q_L = 80\,\mathrm{cm^3\,s^{-1}}$, that is, the superficial Reynolds number,

$$\mathrm{Re}_{SL} = (\rho_L Q_L d)/(A_T \mu_L) = 2250 \qquad [6]$$

where d is the tube diameter, ρ_L is the liquid density and μ_L is the liquid viscosity, the experimental data departs from this relation towards the condition of draining of an initially liquid filled horizontal tube. The point of departure is when turbulent conditions commence and is shown by the arrows of figure 8. Extending the reasoning of Bejamin (1968) it is possible to calculate out the limiting condition of depth given as a dashed line on figure 8.

Figure 9 illustrates the draining condition in a tube filled with a liquid. Normally the air filled cavity or semi-infinite bubble is advancing into the upper section of the tube as the liquid drains away in the opposite direction in the lower part of the tube. In order to

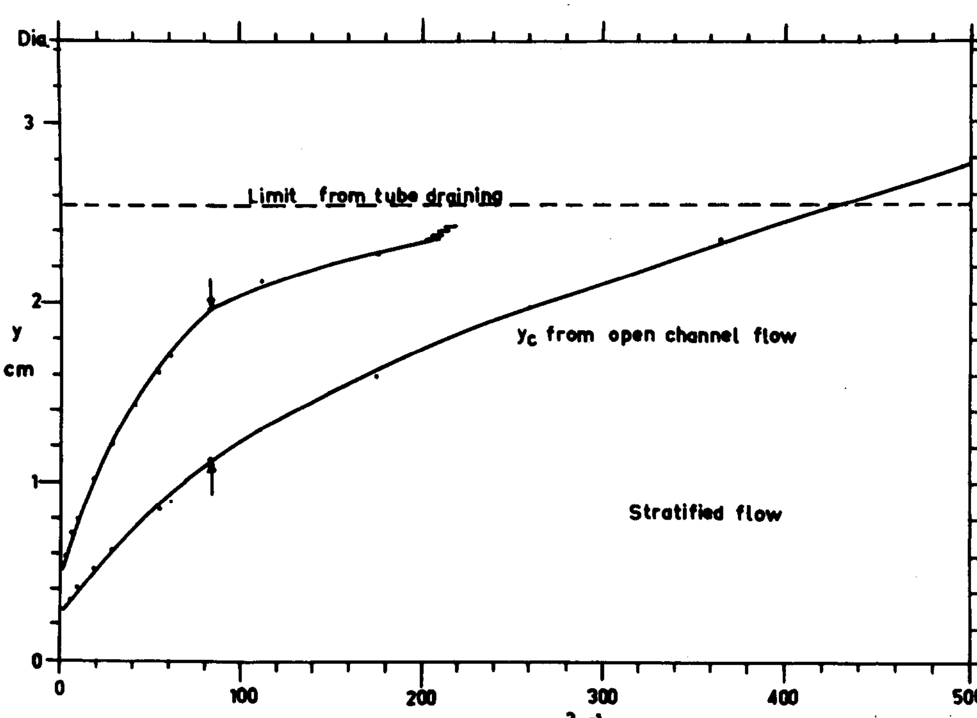

Figure 8. Actual depth of liquid flow against liquid volumetric rate for flow in a horizontal 4.54 cm dia. pipe. Limiting height is taken from tube draining situation. The y_c values are calculated from open channel flow theory.

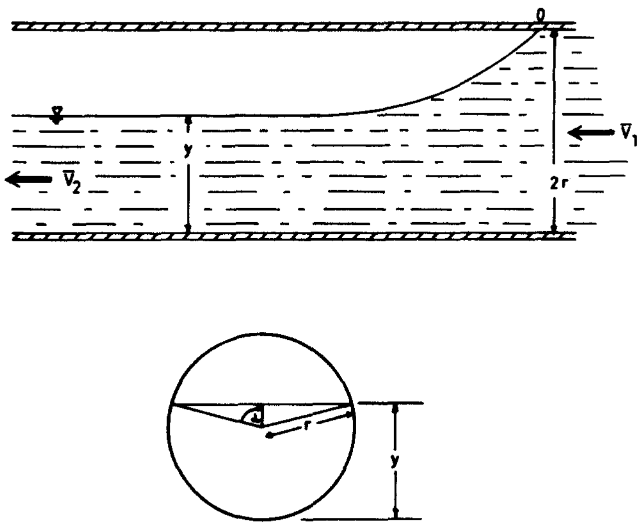

Figure 9. Schematic representation of the draining condition for a horizontal pipe initially filled with liquid.

make the problem more tractable it is assumed that the liquid passes away at such a velocity that the cavity remains stationary with a stagnation point 0 at its tip and a free boundary above the down stream liquid. The cross-sectional area of liquid flow.

$$A_L = (\pi - \alpha + \frac{1}{2}\sin 2\alpha)r^2 = \pi r^2(1 - \xi) \qquad [7]$$

where

$$\xi = \left(\alpha - \frac{1}{2}\sin 2\alpha\right)/\pi \qquad [8]$$

and 2α is the angle subtended at the tube axis by the free surface far downstream, and r is the tube radius.

From the equation of continuity,

$$\bar{V}_1/\bar{V}_2 = A_L/\pi r^2 = 1 - \xi. \qquad [9]$$

Applying Bernoulli's theorem along the free surface between the stagnation point 0 and the asymptotic level of liquid far downstream

$$\bar{V}_2^2 = 2g(r - r\cos\alpha - h) \qquad [10]$$

where h is the head loss due to friction. The pressure at the top of the cross-section for upstream is

$$P_{T_1} = -\frac{1}{2}\rho_L \bar{V}_1^2 \qquad [11]$$

and the pressure in the liquid below has a hydrostatic variation with depth. The total pressure force acting on a cross-section is,

$$F_{P_1} = (P_{T_1} + \rho_L g r)\pi r^2 \qquad [12]$$

while the total flow force must have the momentum flux added

$$F_{f_1} = (P_{T_1} + \rho_L gr + \rho_L \bar{V}_1^2)\pi r^2$$
$$= \rho_L\left(gr \times \frac{1}{2}\bar{V}_1^2\right)\pi r^2. \qquad [13]$$

Downstream the corresponding total flow force is

$$F_{f_2} = 2\rho_L gr^3 \int_\alpha^\pi (\cos\alpha - \cos\theta)\sin^2\theta \, d\theta \qquad [14]$$

$$= \rho_L gr\left(A_L \cos\alpha + \frac{2}{3}r^2 \sin^3\alpha\right) \qquad [15]$$

Since $F_{f_1} = F_{f_2}$ and using [9] and [10] to eliminate the velocity term gives,

$$\xi^2(1 - \cos\alpha) - \frac{2}{3\pi}\sin^3\alpha + \xi \cos\alpha + \frac{h}{r}(+1 - \xi^2) = 0. \qquad [16]$$

Figure 10 details graphs of h/d, $\bar{V}_1^2/(gr)\bar{V}_L^2/(gr)$ and $Q_L^2/(gr^5)$ against y/d.

The calculations were made by first assuming a value of y/d and finding the corresponding values for α and ξ. Solution of [16] under these conditions gives the corresponding value of h/d. The flow force relations [13] and [15] must be equal and since [9] gives the relation between the two velocities it is possible to show that

$$\bar{V}_2^2 = 2gr\left(1 - \cos\alpha - \frac{h}{r}\right) \qquad [17]$$

and

$$\bar{V}_1^2 = 2gr(1 - \xi)^2\left(1 - \cos\alpha - \frac{h}{r}\right). \qquad [18]$$

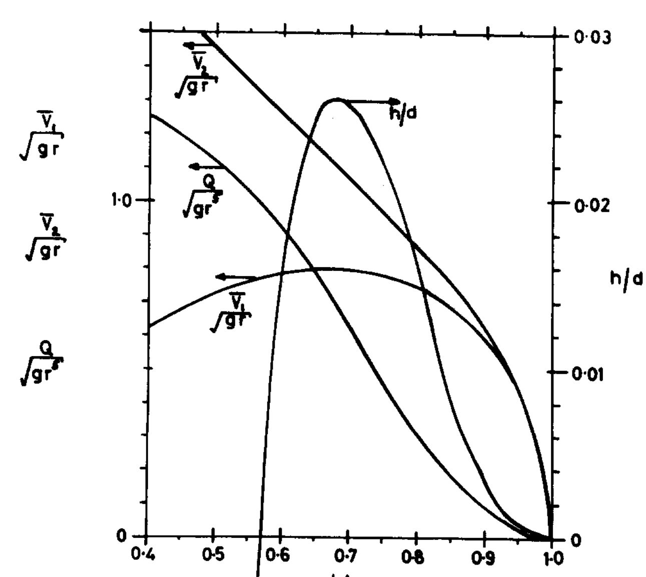

Figure 10. Graphs of various dimensionless parameters found for the free draining condition of a horizontal pipe considered as functions of y/d, the dimensionless liquid depth.

Since the rate of discharge of liquid is equal to the rate of increase of the cavity above the liquid in the tube,

$$Q_L = \bar{V}_1 \pi r^2 \xi \qquad [19]$$

and

$$Q/\sqrt{gr^5} = \pi\xi(1-\xi)\sqrt{2\left(1 - \cos\alpha - \frac{h}{r}\right)} \qquad [20]$$

The inviscid condition applies (i.e. $h = 0$) when $y/d = 0.536$ and the treatment of Benjamin (1968) then is relevant. At $y/d = 0.680$ the values of h/d and $\bar{V}_1^2/(gr)$ are at a maximum showing that the speed of motion of the bubble is directly influenced by the applied forces as reflected by the frictional resistance ratio. Thus when the frictional resistance is positive at $y/d > 0.563$, the flow depth is greater than the minimum value obtained under inviscid conditions because the flow of liquid out of the tube is hindered by the friction. Thus steady flow in which the receding stream fills more than 0.563 d. of the pipe is possible if energy loss occurs. To obtain the condition $h/d < 0.563$, requires the frictional energy loss h to be negative, which would necessitate an external supply of energy to sustain steady flow. As y/d increases from its value of 0.563 at $h = 0$, $Q_L^2/(gd^5)$ falls steadily from the invicid condition. Thus the rate of liquid flow out of the tube cannot be made larger than the value for free flow without energy loss and the only way in which it can be increased is by pumping the liquid in order to overcome the resistance to flow.

The upstream Froude number, $\bar{V}_1^2/(gr)$, first increases with y/d to a maximum value at $y/d = 0.680$ and then steadily falls away. The maximum value of the upstream Froude number coincides with the maximum in the h/d curve in a similar manner as that reported by Benjamin (1968) for a rectangular channel although the exact value was smaller. The form of the $\bar{V}_1^2/(gr)$ against y/d graph shows that within a certain range of y/d values there are two possible values for the downstream depth for each value of upstream velocity. For example, when inviscid conditions pertain the receding liquid stream is supercritical and may be shown to have a Froude number of 1.328 but it is possible for it to acquire another larger depth in the subcritical range by passing through a hydraulic jump. Therefore, steady flow in the range of y/d between 0.563 and about 0.768 would be virtually impossible to maintain particularly close to the latter value since any flow instability, for example, induced by wave formation, would precipitate the hydraulic jump which is latent in the particular conditions. The result would be that the receding liquid flow would commence in the supercritical condition but would soon pass into the alternative depth at subcritical conditions which corresponds to the flow conditions of the free surface established upstream. When the hydraulic jump takes place the interface between the liquid and gas phases not only rises in height but would tend to become blurred due to the onset of gas entrainment in the surface liquid. This would lead readily to a change in flow regime and would explain the blurred region shown on figure 8 when the experimental data are approaching the free draining conditions.

The above working and discussion on the draining condition in a tube initially filled with a liquid, as illustrated in figure 9, obviously leads to a limiting condition for the current two phase flow situation which is being considered in this work. However, the development does have relevance in that it casts some light on the stratified situation under discussion. Firstly for a pumped or gravity fed liquid condition the liquid holdup in the tube will be in general below that of the free draining condition, as suggested by intuitive reasoning earlier. Secondly, departure from inviscid condition can be expected to increase the liquid holdup. Finally as the liquid rate is increased for given conditions the onset of surface disturbances will precipitate flow conditions that cause the flow to depart from the stratified regime before the liquid holdup can reach the free draining situation. Chrisholm & Laird (1958) have demonstrated that both liquid holdup and two phase pressure drop increase with tube roughness in agreement with the second prediction. The data of figure 8 show that as the liquid rate is increased the liquid holdup does approach the free draining condition but the flow changes into the mixed type regime before the actual data can coincide with the theoretically derived free draining condition.

Vertical upward flow

Data for vertical upwards two-phase flow in pipes are presented in figures 3, 11–13 where they are compared with the correlations suggested in this work for horizontal flow. The air–water data of Spedding & Nguyen (1976) obtained for a 4.54 cm i.d. tube are shown in figure 11 and exhibit a series of curves which possess a systematic variation with superficial liquid velocity. In general the curves are not smooth for values of superficial liquid velocity, $\bar{V}_{SL} < 0.28\ \mathrm{m\,s^{-1}}$ but exhibit a number of discontinuities which correspond to changes in flow regime. At low values of Q_G/Q_L the flow regimes are the bubble, slug and slug plus froth types. At point A, for example, on figure 11 there is an abrupt change in slope when the flow regime passes to the annular plus wave type of flow. Again, at point B the flow becomes annular while at point C the curve has a tendency to level off as the droplet type of flow commences. Similar rather abrupt changes in slope were noted in the horizontal case but of course there was no complication of a variation with superficial liquid velocity. Conversely with vertical upwards flow there is no complication from the stratified regime since it does not occur for the case of vertical upwards flow.

Thus a correlation for the case of vertical upwards flow, which is developed using the two parameters of holdup and volume flow ratios, is complex and cannot readily be reduced to a simple form of the type which have been obtained for the case of horizontal flow. The variation with superficial liquid velocity which has been mentioned can be accommodated by the two relations,

$$\bar{R}_G/\bar{R}_L = 1/[0.2 + K_1 Q_L/Q_G] \qquad [21]$$

for the region $\bar{R}_G/\bar{R}_L \leqslant 4.0$ where the Armand relation applied for horizontal flow case, and

$$\bar{R}_G/\bar{R}_L = K_2[1 - \exp[-K_3 Q_G/Q_L]]\,[Q_G/Q_L]^{0.65} \qquad [22]$$

for the region $\bar{R}_G/\bar{R}_L > 4.0$–275. The relations

$$\ln(K_1) = -1.44\ln(\bar{V}_{SL}) - 0.007 \qquad [23]$$

$$K_2 = 0.14\ln(\bar{V}_{SL}) + 1.0 \qquad [24]$$

$$\ln(K_3) = 0.97\ln(\bar{V}_{SL}) - 3.0 \qquad [25]$$

provide the observed variation with $\bar{V}_{SL}$. The values of $K_{1,2,3}$ are made dimensionless by the appropriate choice of units for the numerical constant in (23)–(25). In addition K_1 has an upper limit of about 50 and a definite lower limit of 1.2 corresponding to the Armand relation which is obtained at values of $\bar{V}_{SL} > 1.0\ \mathrm{m\,s^{-1}}$. Also K_2 and K_3 have lower limits of about 0.25 and 0.003 and definite upper limits of 0.68 and 0.0057, respectively. Of course, as the value of Q_G/Q_L increases to somewhat beyond 10^4, the value of $\bar{R}_G/\bar{R}_L$ at first becomes constant at about 275 for this case and then increases rapidly to the homogeneous line where $\bar{R}_G/\bar{R}_L = Q_G/Q_L$.

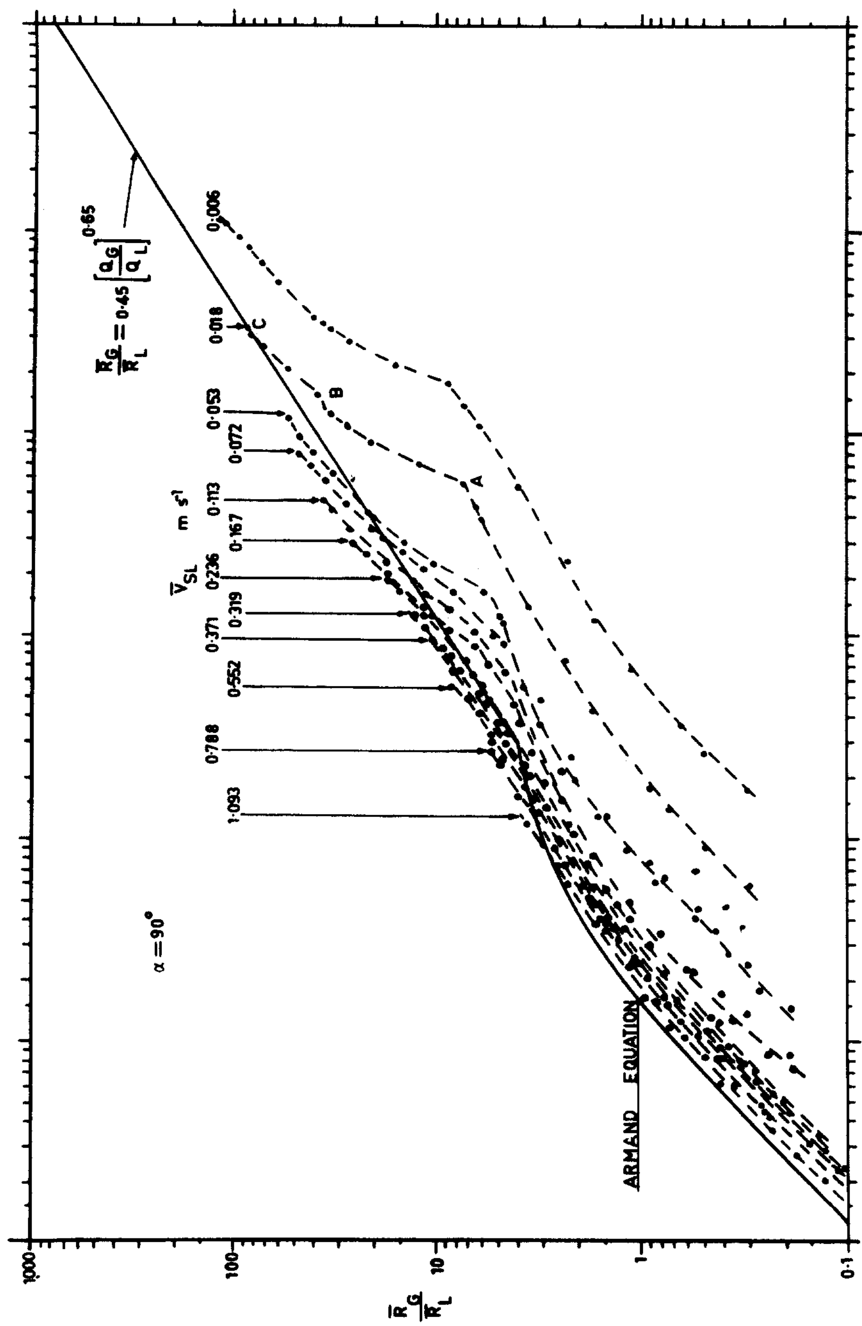

Figure 11. Vertical two-phase holdup data for air–water flow for a 4.54 cm i.d. pipe. $\bar{V}_{SL}$ = superficial liquid velocity.

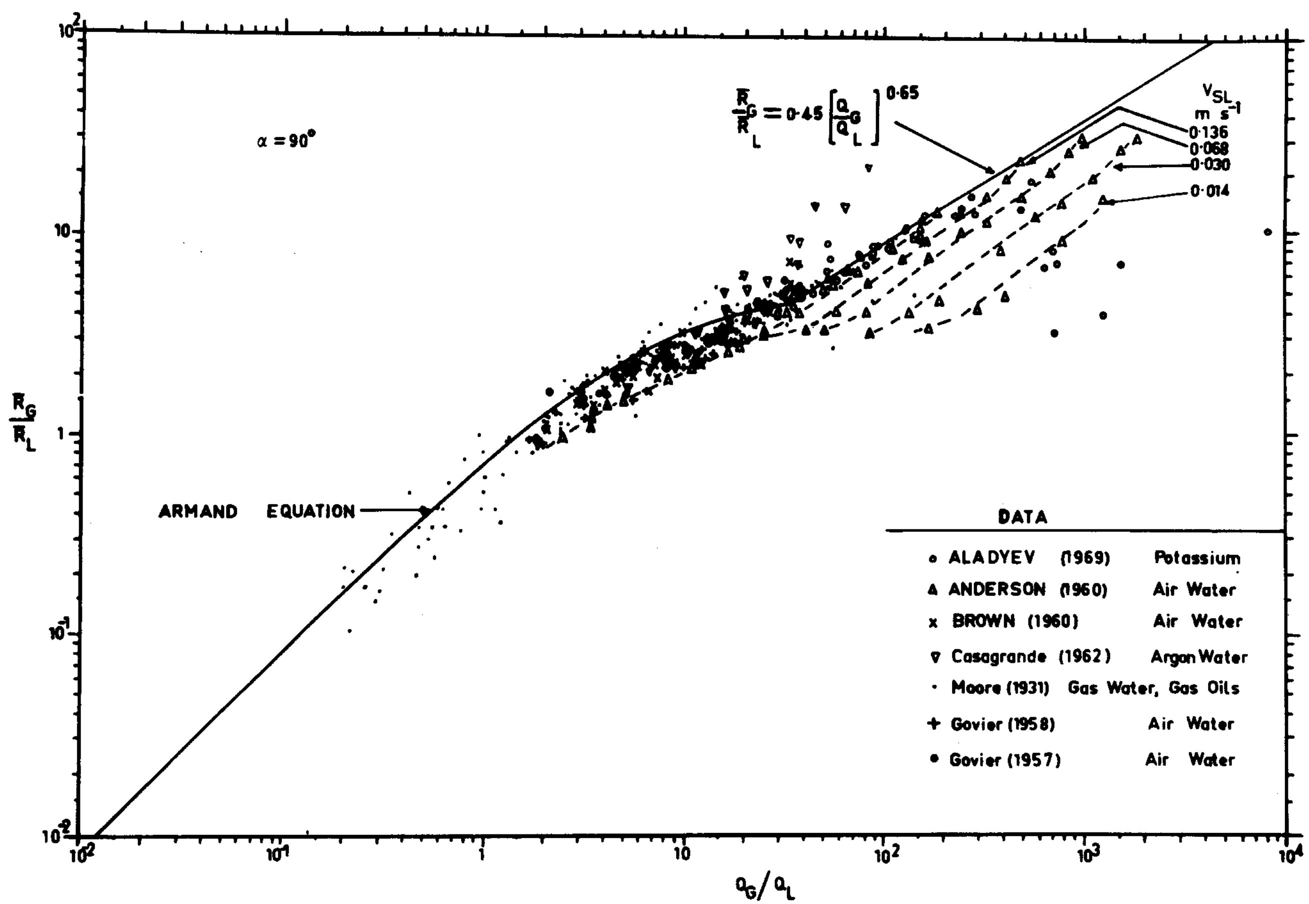

Figure 12. Vertical two-phase holdup data for various different systems and the correlations suggested in this work. $\bar{V}_{SL}$ = superficial liquid velocity.

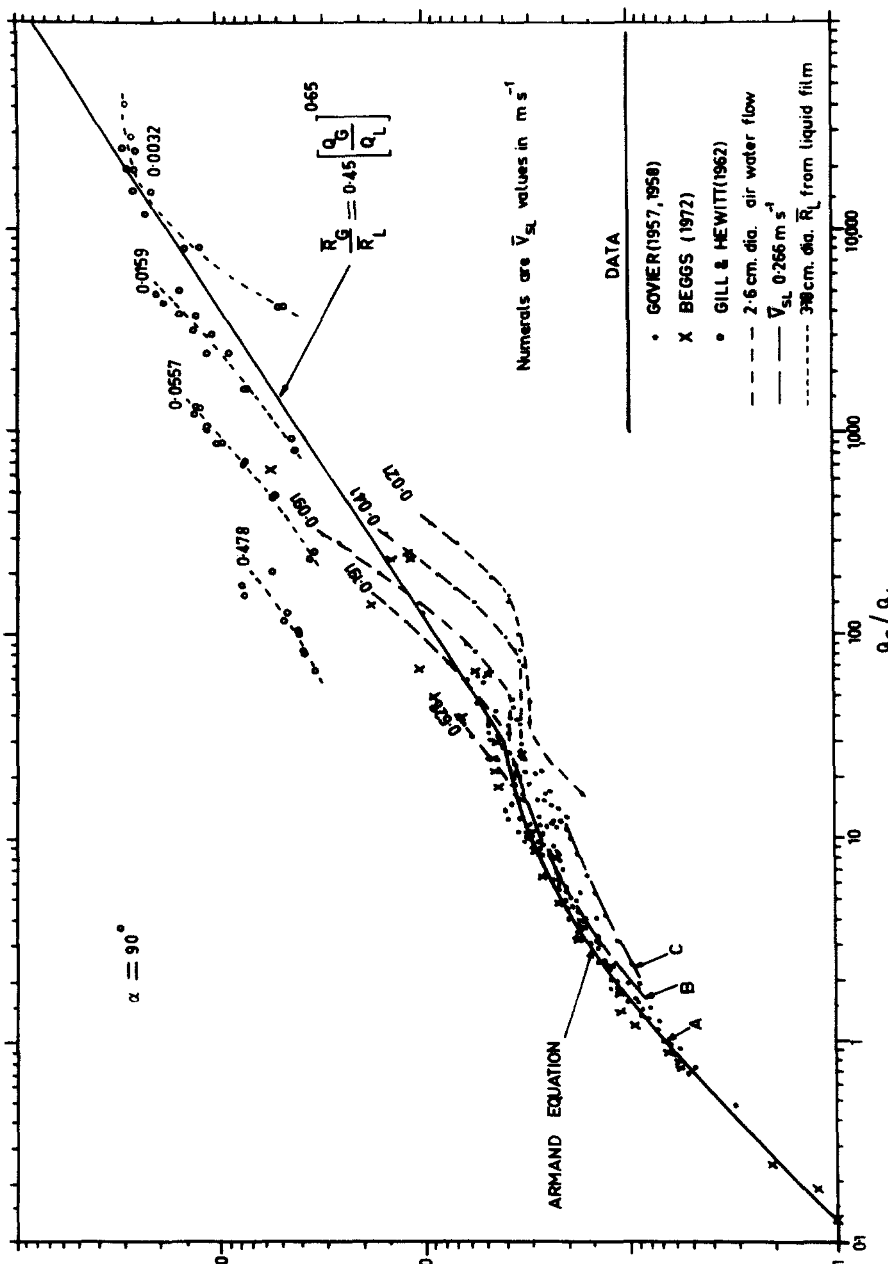

Figure 13. Vertical two-phase holdup data for air–water reported by various workers and the correlations suggested in this work. $\bar{V}_{SL}$ = superficial liquid velocity.

In figures 12 and 13, data from various literature sources are plotted. The vertical flow data of Aladyev *et al.* (1969) were obtained for potassium two phase flow in a tube of diameter range between 0.54 and 0.625 cm. The data give good agreement with figure 11 up until $Q_G/Q_L \lesssim 300$. Beyond this point significant departure is observed as the $\bar{R}_G/\bar{R}_L$ value fell away. The trend is opposite to that expected from $\bar{V}_{SL}$ variation but can be traced to the electrical resistance method which was used to measure holdup. The technique operated satisfactorily until the point was reached when the flow regime changed from pure annular flow into droplet plus annular flow. This latter was a flow regime situation for which the apparatus was not calibrated and therefore errors were obtained. Most of the other data given in figure 12 were obtained at $\bar{V}_{SL} \gg 0.15\ \mathrm{m\ s^{-1}}$ and therefore did not show any effect of superficial liquid velocity. The expection was the data of Anderson & Mantzouranis (1960) for 1.08 cm dia. vertical pipe where a definite effect of liquid velocity was observed which roughly corresponded to that presented in figure 11. The data of Govier *et al.* (1957) and Govier & Short (1958) for 2.6 cm dia. pipe and Brown *et al.* (1960) for 3.81 cm dia. tube gave reasonable agreement with that of figure 11. The Argon–water data of Casagrande *et al.* (1962) for 2.5 cm dia. tube gave reasonable agreement up to Q_G/Q_L of 10 and thereafter showed considerable departure from figure 11 data. The data of Moore & Wilde (1931) for gas–water, gas oil, kerosene and two types of light lubricating oils over a pipe size range from 2.54 to 9.6 cm, were scattered and, being obviously very inaccurate below Q_G/Q_L of 0.2, that part of the data were omitted from the plot. There did not appear to be any effect of diameter on the results but the liquid holdup was observed to increase as the viscosity of the liquid phase rose for the heavier of the lubricating oils. Such an increase is to be expected. Spedding *et al.* (1982) have pointed out that above a liquid viscosity of $2.0–3.0 \times 10^{-2}\ \mathrm{kg\ m^{-1}\ s^{-1}}$ the pressure loss will increase substantially above that for the air–water system.

In figure 13, detailed results from the U.S. documentation centre of Govier *et al.* (1957) and Govier & Short (1958) are presented. The former show an effect of $\bar{V}_{SL}$ which approximately parallels the data in figure 11, while the latter indicate an effect of diameter at const. $\bar{V}_{SL}$. On the figure 13, curve A is for 2.60 cm dia. tube and under, curve B is for 3.81 cm dia. tube and curve C is for 6.35 cm dia. tube. It is observed, therefore, that the liquid holdup increases somewhat with tube diameter. In figure 3 no such effect is given for the annular steam–water data of Harrison (1975) at high liquid flow rates for 20 cm dia. pipe. In addition the data of Moore & Wilde (1931), which is admittedly scattered, does not highlight any diameter effect over a wider range of geometry. However, the slug flow data of Govier & Short (1958) exhibit consistent trends which usually accompany reliable results and therefore it can be presumed that there is an effect of diameter in the slug flow regime. The limited slug flow data of Lupoli *et al.* (1973) would add weight to this conclusion. This would mean that the relation given by [21] only applies for a diameter under about 5 cm while a relation given by [22] possibly is of general application.

The data of Gill & Hewitt (1962) which are included in figure 13 were obtained from film thickness measurements and since the droplet phase is excluded are considerably at variance with other work examined here. However, the data do exhibit the general trend with superficial liquid velocity which has been observed in figure 11 as well as the tendency for the holdup ratio to come to a constant value at Q_G/Q_L greater than 10^4.

Other workers have reported on holdup measurement in vertical two phase flow but in most cases the data were not in a form to allow them to be used for this comparison (see Hughmark & Pressburg 1961; Ros 1961; Cravarola & Hassid 1965; Ueda 1967; Ellis & Lloyd Jones 1965; Yamazaki & Shiba 1969; Oshinowo & Charles 1974; and Yamazaki & Yamaguchi 1967).

Downward flow

The data for the downward angles of flow are detailed in figures 14–18, together with what data are available from the work of Beggs (1972). The limitations of these latter data have been mentioned previously. In the interests of clarity it has not been possible to label each cross on the diagram with the actual superficial liquid velocity values. However, in general, the data give reasonable agreement with the curves on figures 14–18 despite the need for interpolation. It is immediately obvious that there is an increase in gas holdup for a given set of conditions compared with the horizontal flow data of figure 2. In essence this means that the liquid flows through the pipe more readily if downward inclinations are employed. For example, comparison between the horizontal data of figure 2 and the $-6.17°$ angle data of figure 14 shows that there is approximately a ten fold increase in holdup ratio $\bar{R}_G/\bar{R}_L$ for similar conditions if the pipe is inclined downwards. Since two phase flow pipe lines are never exactly level but must be sloped slightly to facilitate drainage it is important to ensure that the fall designed into the system is in the direction of fluid movement in order to enable the liquid to be handled more readily.

Again using the parallel with open channel flow it is possible to modify the development of [3] and [4] by using the concept of total energy H as applied to figure 19.

$$y_1/\cos\alpha + \frac{\bar{V}_1^2}{2g\cos\alpha} + l\sin\alpha = y_2/\cos\alpha + \frac{\bar{V}_2^2}{2g\cos\alpha} \qquad [26]$$

$$k_1 + \frac{\bar{V}_1^2}{2g\cos\alpha} + \Delta Z = k_2 + \frac{\bar{V}_2^2}{2g\cos\alpha} = H. \qquad [27]$$

Since H is constant then for the case of inviscid flow in general,

$$(H-k)A_L^2 = \frac{Q_L^2}{2g\cos\alpha} \qquad [28]$$

which gives a cubic relation of the type shown in figure 20, the method of solution of which is given in the appendix. The form of the relation is similar to that given by [4] in figure 6 but [28] gives different values of the critical depth y_c as shown in figure 21 for $\alpha = -6.17°$. The major point to notice is that the actual depth is below the critical depth so that the flow is supercritical with the $Fr > 1.0$. In general, the depth of flow is approximately half the critical depth and falls as the angles is increased. The speed of the flow is such that disturbances which may manifest themselves as surface waves will be carried downstream with the liquid flow thus tending to stabilise the flow into the stratified regime.

The Manning equation is used to express the flow in inclined open channels,

$$Q_L = [A_L(\bar{r})^{2/3}S^{1/2}]/n \qquad [29]$$

where $\bar{r}$ is the hydraulic mean radius being the area of liquid flow A_L over the wetted perimeter of the flow channel, S is the slope of the channel and n the Manning roughness coefficient of $0.0095 \times 3.2805 = 0.0312$ m. Figure 21 shows that the Manning equation accurately predicts the actual experimental holdup for the case of $\alpha = -6.17°$. Table 1 gives the critical depth and the actual experimental depth together with the Manning equation predictions for various other downward angles of inclination. As far as the Manning equation is concerned it gave excellent agreement with experimental holdup data for low liquid flow rates at low angles of inclination but appreciable departure occurred as these two parameters increased in magnitude.

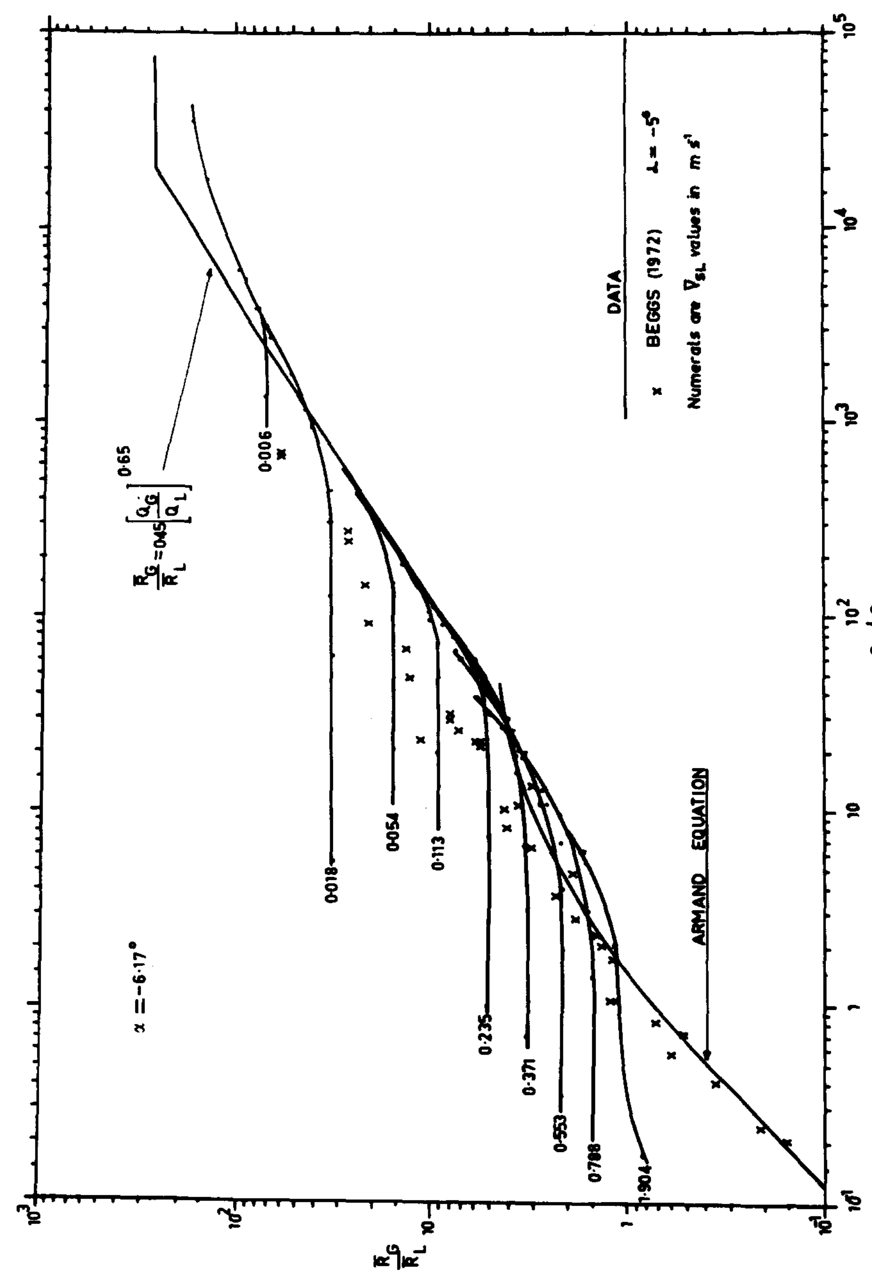

Figure 14. Two-phase flow air–water holdup data for a 4.54 cm i.d. pipe at an angle of $\alpha = -6.17°$.
$\bar{V}_{SL}$ = superficial liquid velocity.

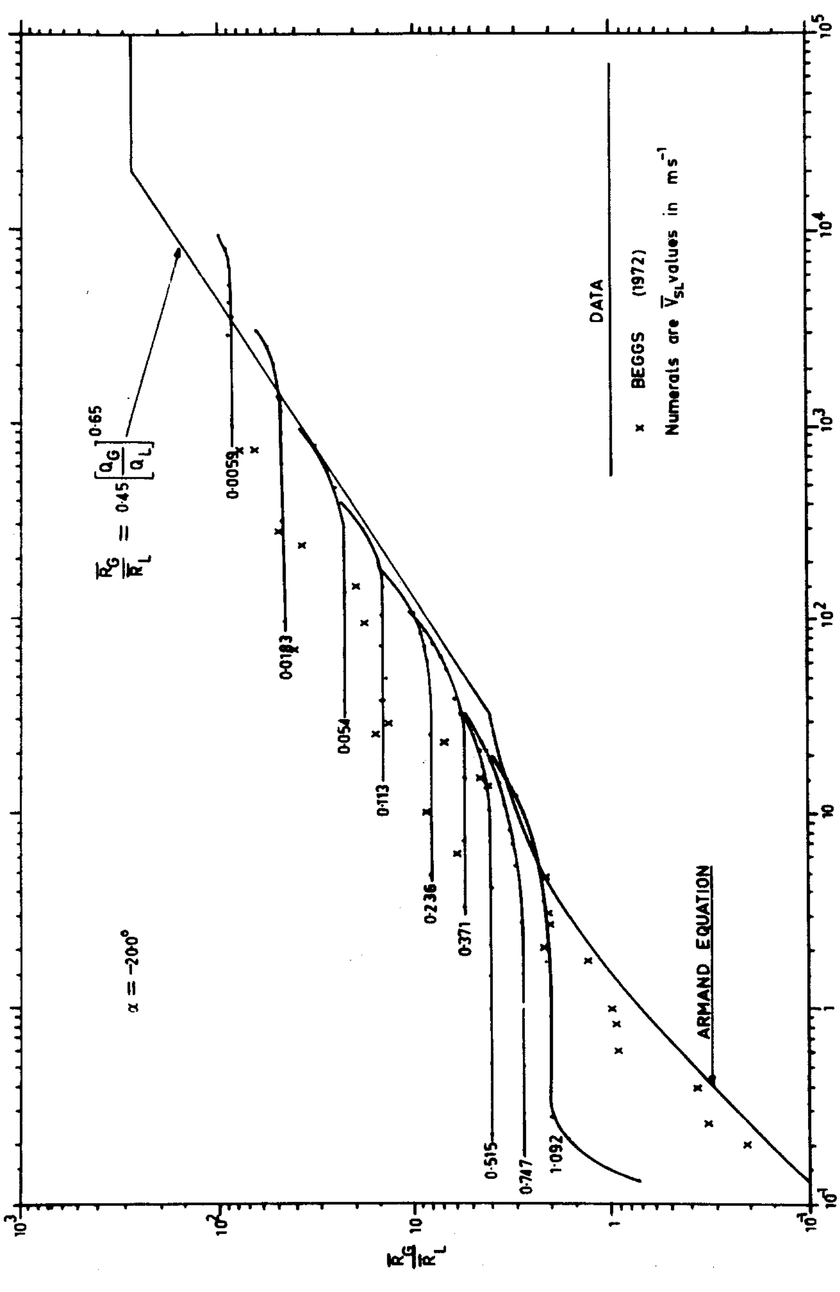

Figure 15. Two-phase flow air–water holdup data for a 4.54 cm i.d. pipe at an angle of $\alpha = -20.0°$. $\bar{V}_{SL}$ = superficial liquid velocity.

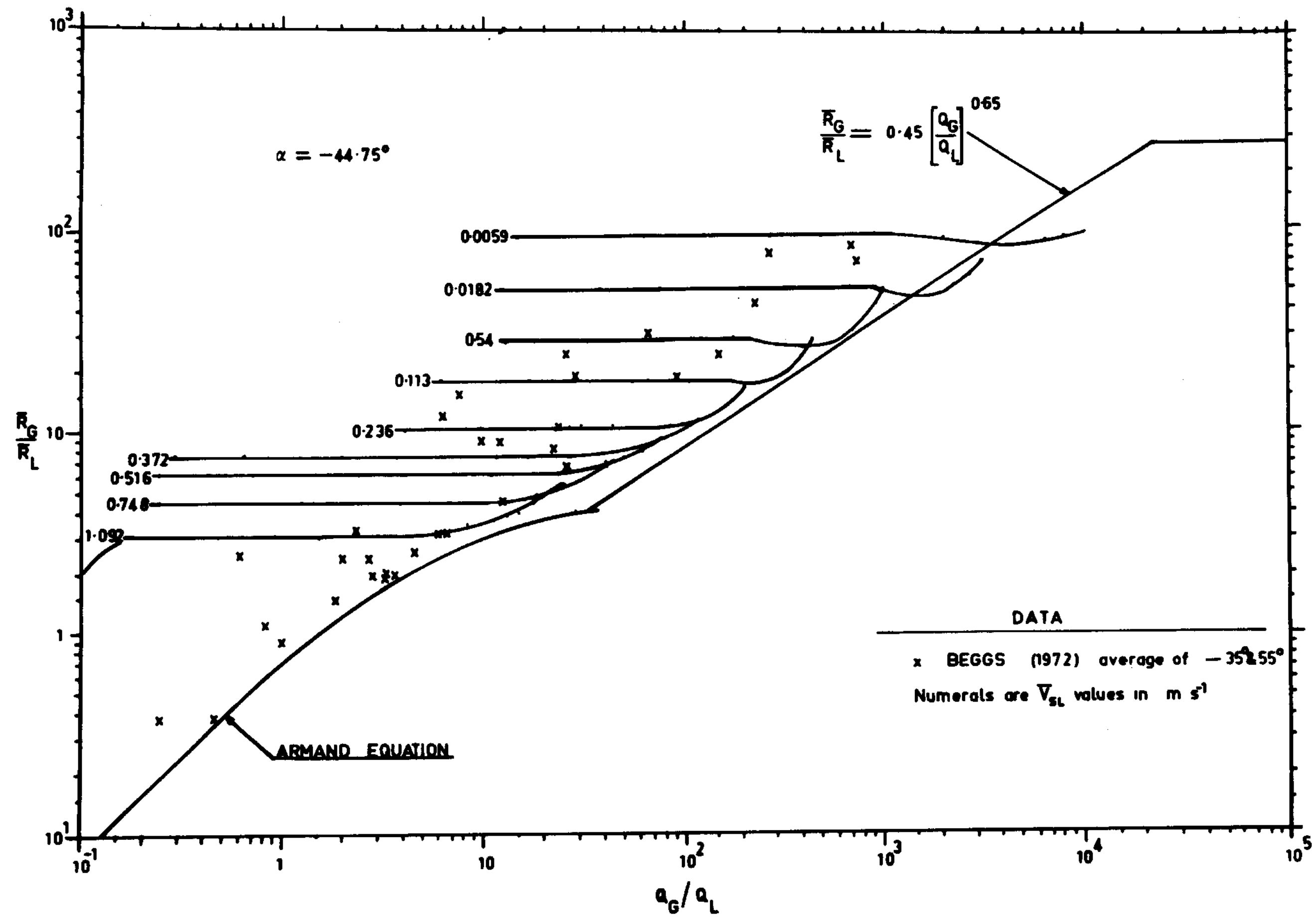

Figure 16. Two-phase flow air–water holdup data for a 4.54 cm i.d. pipe at an angle of $\alpha = -44.75°$. $\bar{V}_{SL}$ = superficial liquid velocity.

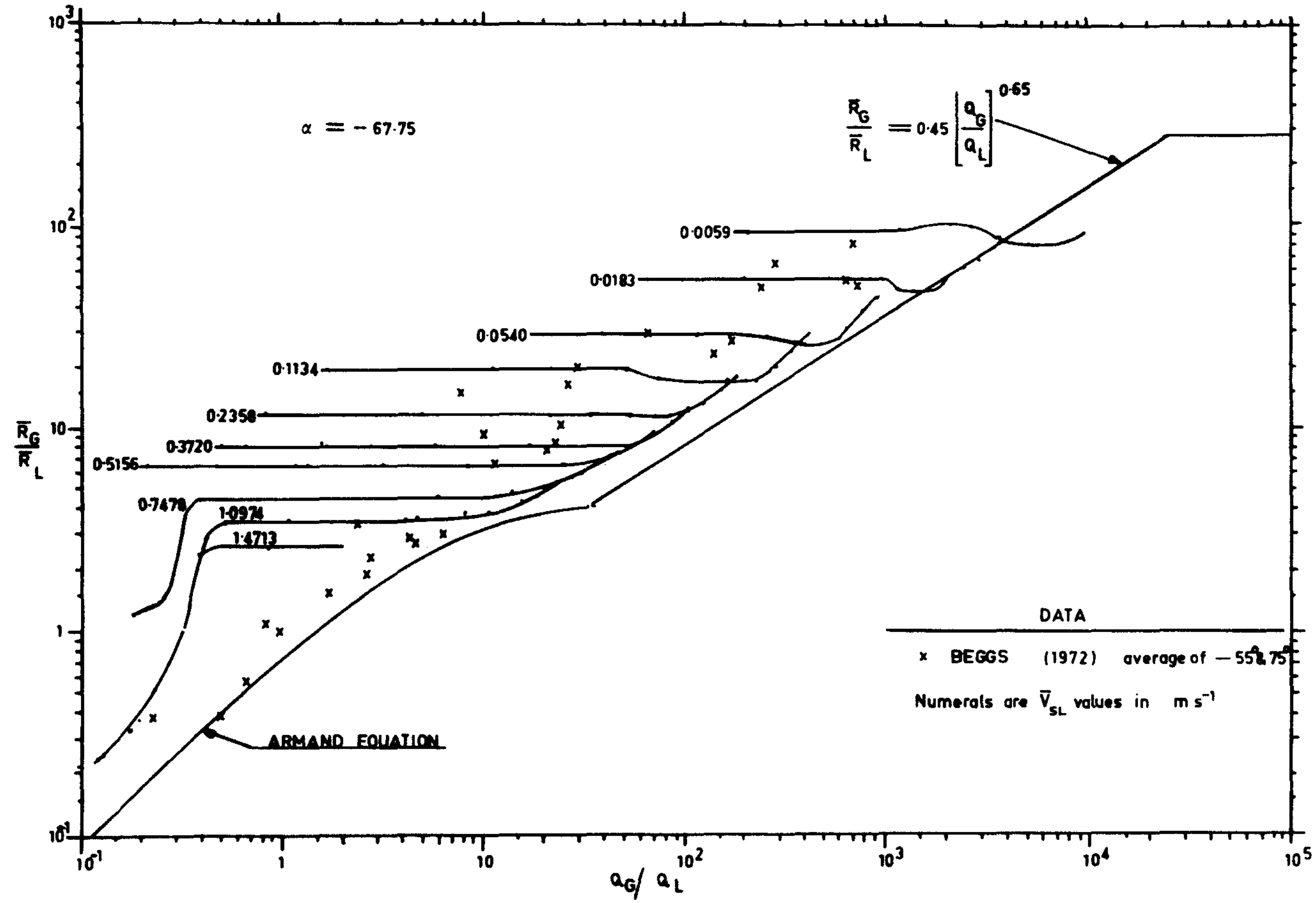

Figure 17. Two-phase flow air–water holdup data for a 4.54 cm i.d. pipe at an angle of $\alpha = -67.75°$. $\bar{V}_{SL}$ = superficial liquid velocity.

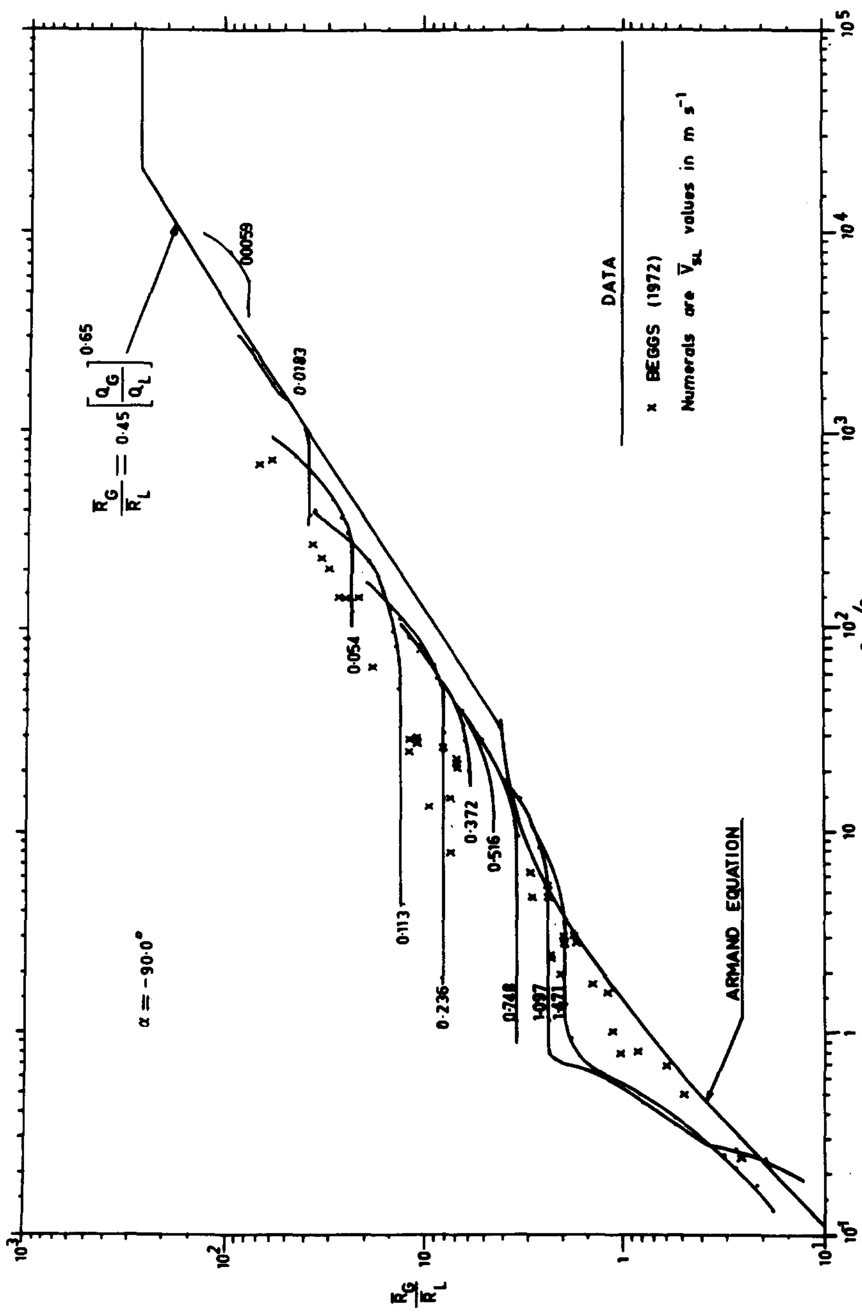

Figure 18. Two-phase flow air–water holdup data for a 4.54 cm i.d. pipe for vertical downward flow $\alpha = -90°$. $\bar{V}_{SL}$ = superficial liquid velocity.

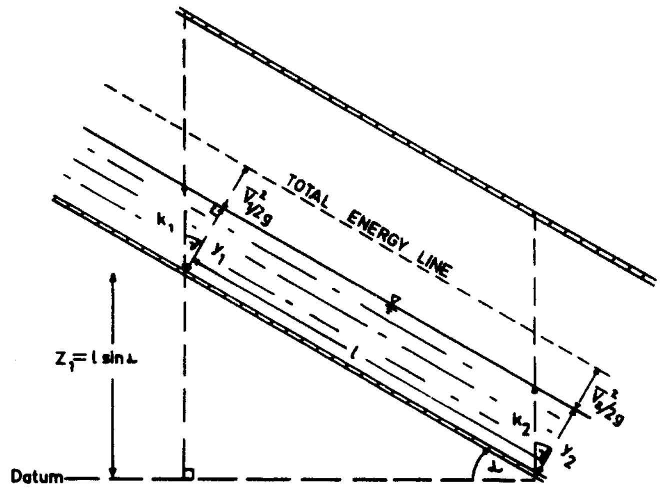

Figure 19. Schematic representation of free surface channel flow in a downward inclined circular conduit. $\bar{V}$ = velocity of liquid flow.

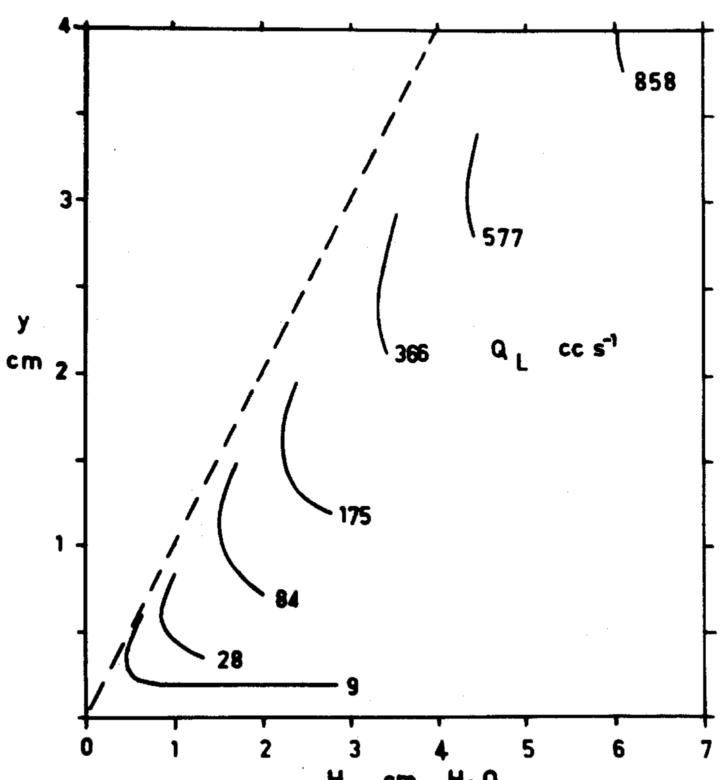

Figure 20. Calculated liquid depth against specific energy for the free surface channel flow in a downward inclined circular conduit 4.54 cm dia. at $\alpha = -6.17°$. Q_L = liquid volumetric flow rate.

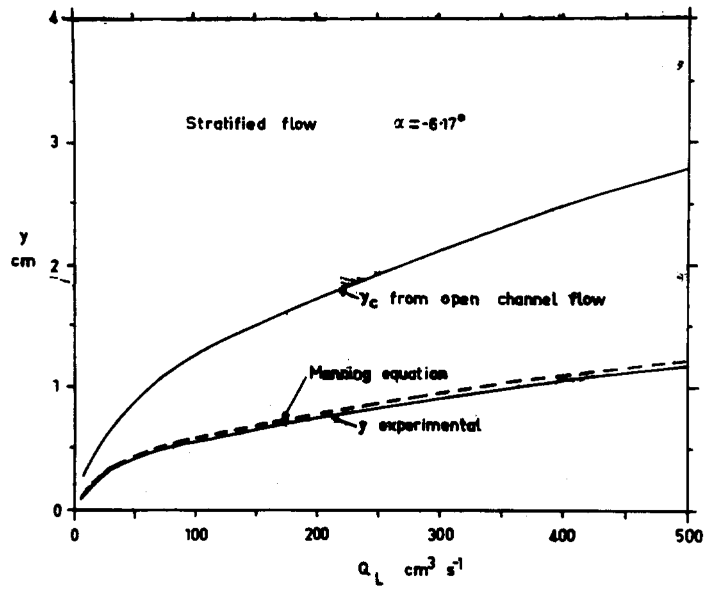

Figure 21. Depth of liquid flow against liquid volumetric flow rate for flow in a downward inclined circular conduit of 4.54 cm dia. at $\alpha = -6.17°$.

Inclined upwards flow

Data for inclined upwards flow are shown in figures 22–25 and possess similar overall trends to that for vertical upwards flow. Accordingly the data are expressed in the same form as [21]–[25] with the details of the actual constants being as shown in table 2.

CONCLUSIONS

Experimental data available from the literature for different pipe inclinations (Isbin 1957, 1959; Fujie 1964; Govier & Omer 1962; etc.) were shown to be correlated by the general correlation method of Chen & Spedding (1983) for the bubble, slug and annular flow regimes. Horizontal stratified flow data exhibits constant $(\bar{R}_G/\bar{R}_L)$ values for low values of Q_G/Q_L. Examined in terms of the Bernoulli equation, it was found that the liquid depth corresponded to approximately twice the critical depth when the liquid rate is below a certain value, but approaches the condition of draining of an initially liquid filled horizontal tube, when this value of liquid rate is exceeded. The analysis showed that the liquid holdup cannot in general be greater than that of the free draining condition although departure from inviscid conditions can be expected to increase the holdup, and as such was shown to be a special case of the draining of a liquid filled tube.

Data for vertically upward and upward inclined flows were shown to fall about a series of lines having similar shape as, but slightly displaced from, those given by Chen & Spedding (1983) for horizontal case. Hence, correlation factors were incorporated to describe these conditions.

Data for inclined downward flow showed that for the same flow situations, compared with horizontal flow, there is a higher gas holdup indicating a higher liquid velocity. Again, the Bernoulli equation was applied treating the situation as open channel flow. It is shown that liquid depths are always below the critical depth with $Fr > 1.0$ and the depth of flow in general being approximately half the critical depth. The depth of flow falls with the increasing angle of inclination. For small angles of inclination, the Manning equation was found to be adequate in describing the liquid depth corresponding to the holdup.

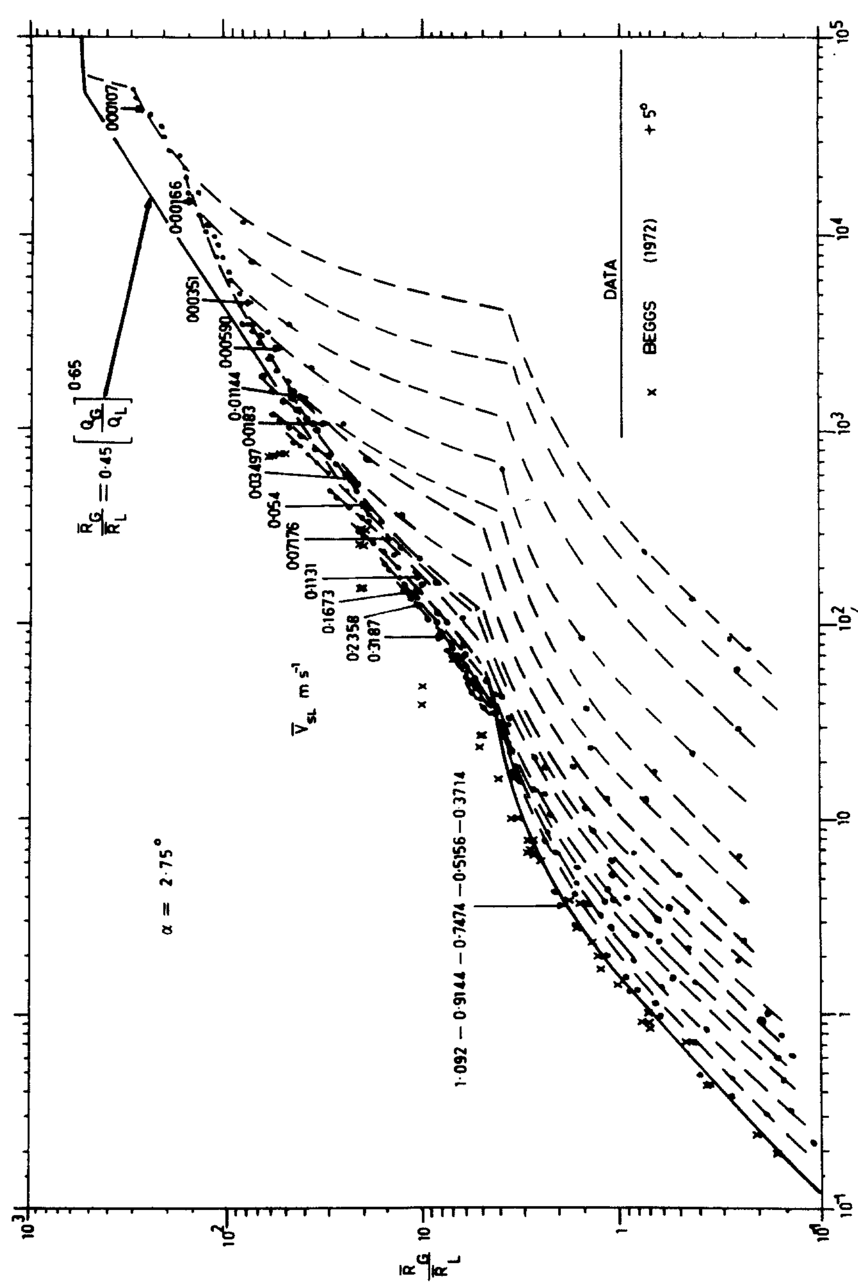

Figure 22. Two-phase flow air–water holdup data for a 4.54 cm i.d. pipe in upward flow at an angle of + 2.75. $\bar{V}_{SL}$ = superficial liquid velocity.

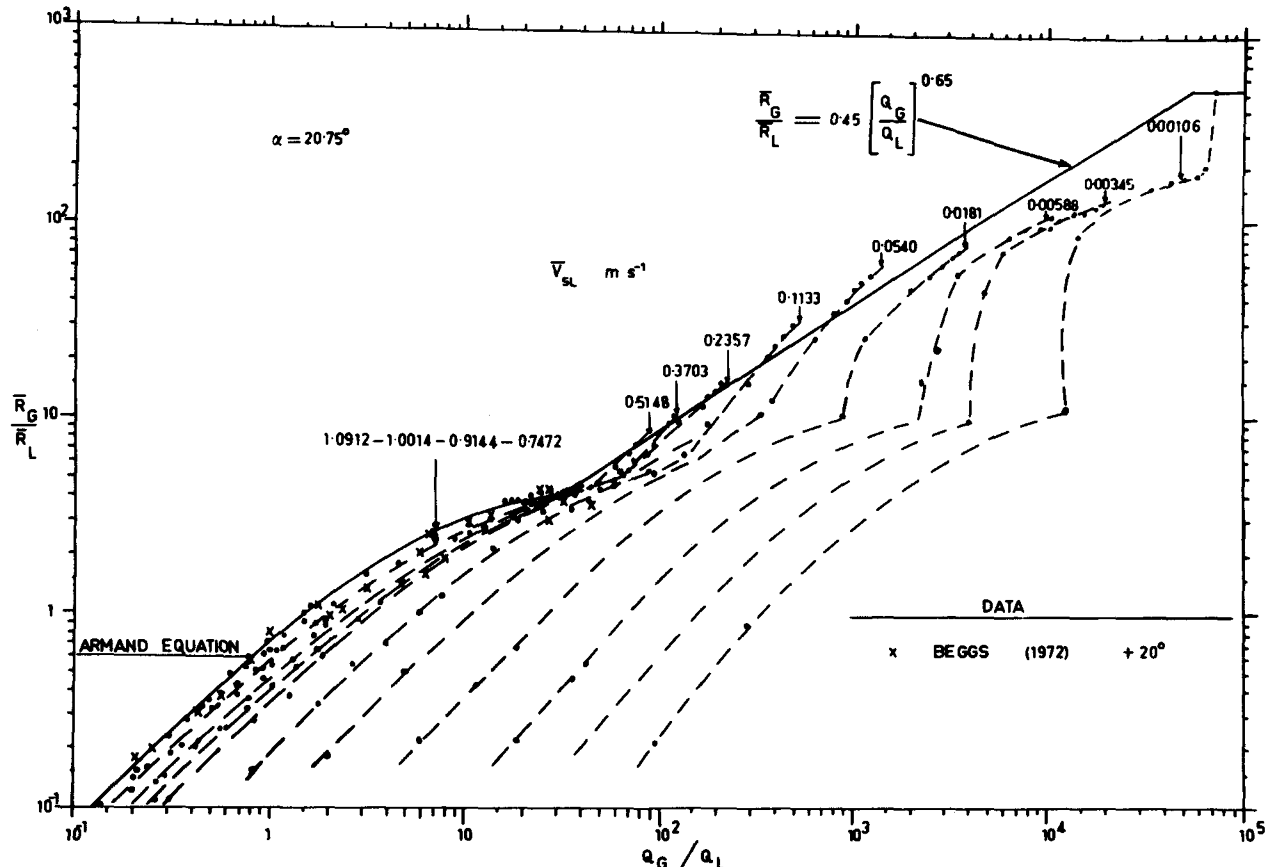

Figure 23. Two-phase flow air–water holdup data for a 4.54 cm i.d. pipe in upward flow at an angle of + 20.75°. $\bar{V}_{SL}$ = superficial liquid velocity.

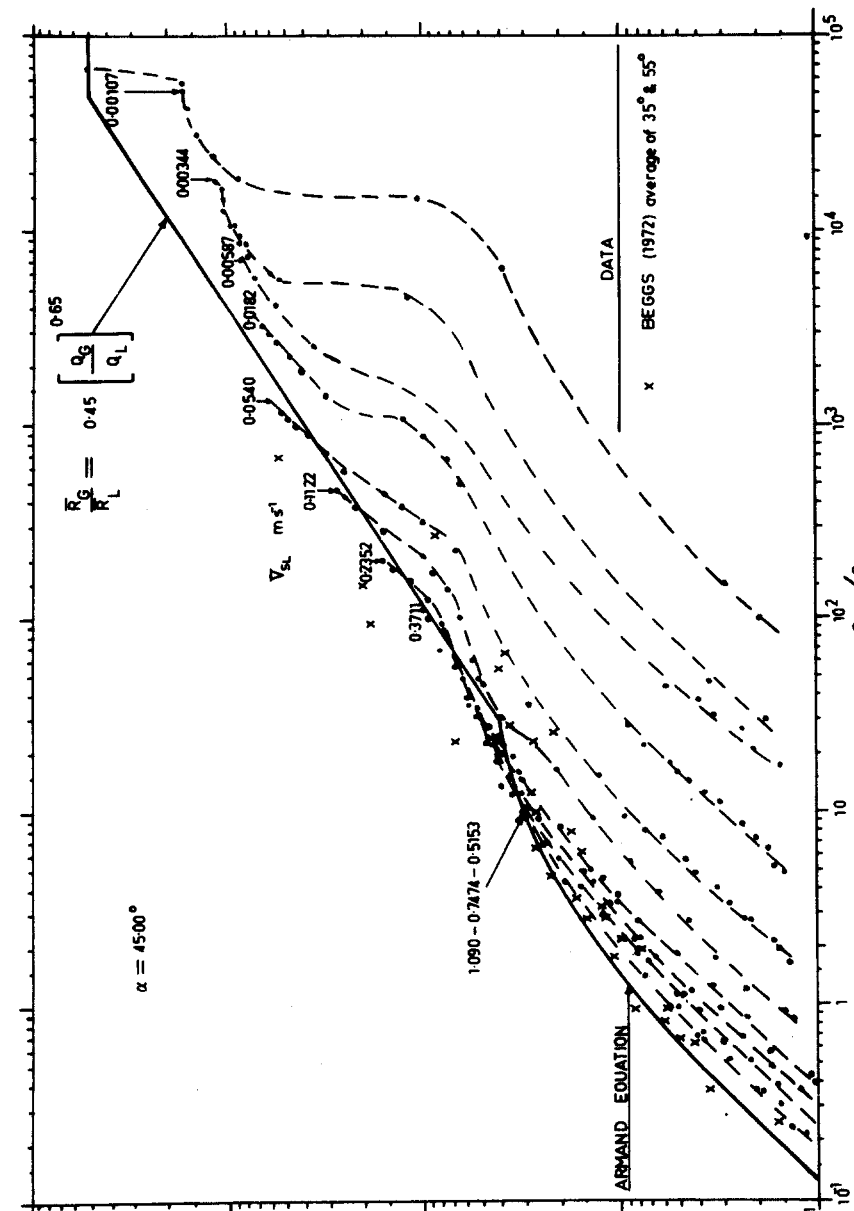

Figure 24. Two-phase flow air–water holdup data for a 4.54 cm i.d. pipe in upward flow at an angle of +45°. $\bar{V}_{SL}$ = superficial liquid velocity.

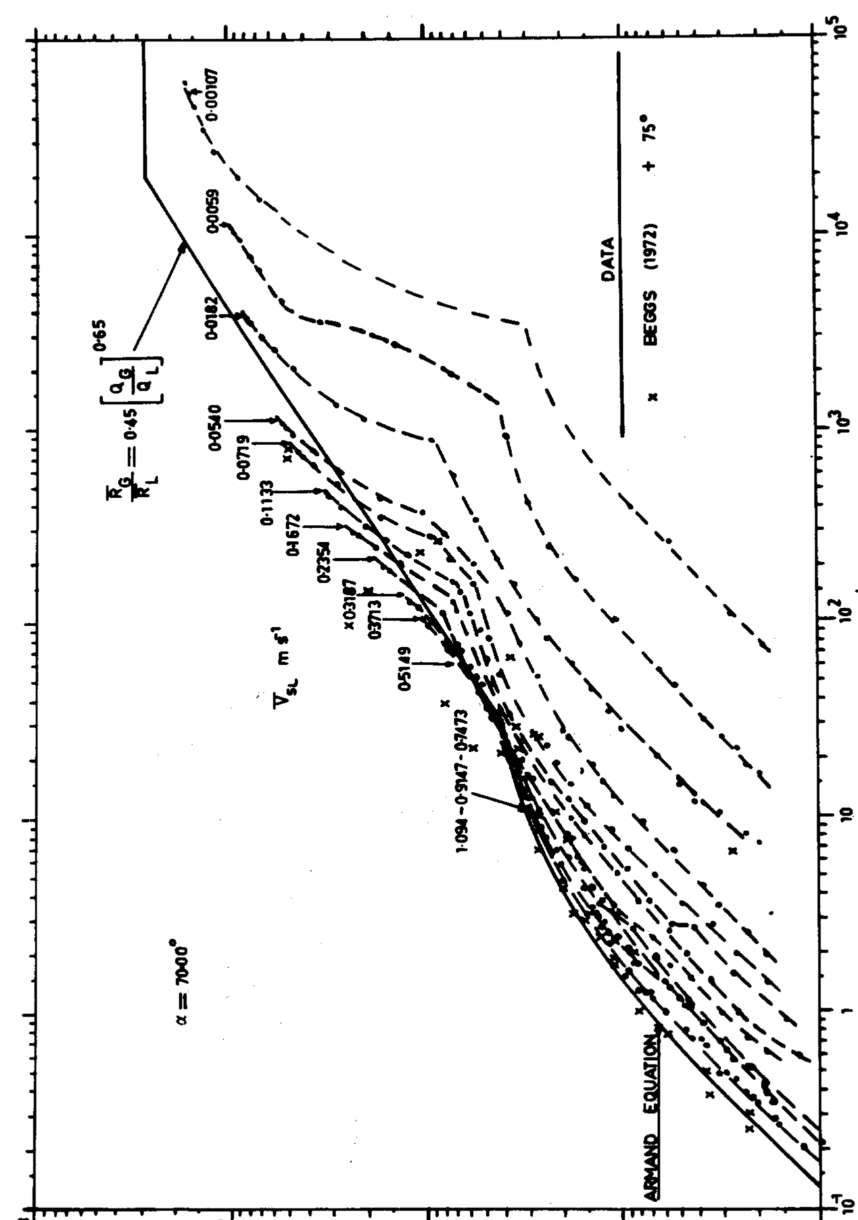

Figure 25. Two-phase flow air–water holdup data for a 4.54 cm i.d. pipe in upward flow at an angle of + 70°. $\bar{V}_{SL}$ = superficial liquid velocity.

Table 1. Depth of liquid in cm in a 4.54 cm dia. tube for various angles of inclination

Q_L	Critical Depth y_c				Experimental Depth y				Manning Equation (29)			
cc s^{-1}	-6.17^o	-20.0^o	-44.75	-67.75	-6.17^o	-20.00^o	-44.75^o	-67.75	-6.17^o	-20.0^o	-44.75^o	-67.75
9.1	0.34	0.36	0.36	0.36	0.185	0.161	0.151	0.151	0.19	0.14	0.11	0.09
28	0.62	0.63	0.64	0.63	0.316	0.244	0.223	0.212	0.32	0.24	0.19	0.15
84	1.14	1.14	1.16	1.16	0.494	0.384	0.341	0.331	0.54	0.40	0.31	0.26
176	1.60	1.60	1.61	1.62	0.684	0.509	0.455	0.447	0.77	0.57	0.45	0.36
366	2.35	2.35	2.37	2.37	1.030	0.730	0.630	0.630	1.10	0.82	0.64	0.51
577	3.03	3.02	2.99	2.99	1.270	0.977	0.782	0.770	1.39	1.02	0.80	0.64
801	-	3.79	3.56	3.54	-	1.083	0.896	0.870	-	1.21	0.94	0.75
858	3.96	3.97	-	-	1.56	1.156	-	-	1.71	1.25	-	-
1161	-	4.11	4.10	4.09	-	1.258	1.073	1.065	-	1.46	1.13	0.90
1224	-	4.15	-	-	1.90	1.378	-	-	2.09	1.50	-	-
1698	-	4.45	4.32	4.31	2.27	1.623	1.315	1.243	2.53	1.78	1.37	1.09
2287	-	-	-	4.54	-	-	-	1.490	-	-	-	1.27

Table 2. Correlations of upward inclined flow holdup data using the K relations of [21]–[25]

	Relations with $\bar{V}_{SL} = m\ s^{-1}$	Angle
Equation (23)	$\ln(K_1) = -1.44 \ln(\bar{V}_{SL}) - 0.007$	$+90^{o}$
	$\ln(K_1) = -0.93 \ln(\bar{V}_{SL}) - 0.44$	$+70^{o}$
	$\ln(K_1) = -0.93 \ln(\bar{V}_{SL}) - 0.26$	$+45^{o}$ & 20.75^{o}
	$\ln(K_1) = -0.91 \ln(\bar{V}_{SL}) - 0.60$	$+2.75^{o}$
Equation (24)	$K_2 = 0.14 \ln(\bar{V}_{SL}) + 1.0$	$+90^{o}$ & 70^{o}
	$K_2 = 0.072 \ln(\bar{V}_{SL}) + 0.66$	$+45^{o}$ & 20.75^{o}
	$K_2 = 0.072 \ln(\bar{V}_{SL}) + 0.74$	$+2.75^{o}$
Equation (25)	$\ln(K_3) = 0.97 \ln(\bar{V}_{SL}) - 3.0$	$+90^{o}$
	$\ln(K_3) = 0.96 \ln(\bar{V}_{SL}) - 3.2$	$+70^{o}$
	$\ln(K_3) = 0.88 \ln(\bar{V}_{SL}) - 3.2$	$+45^{o}$, 20.75^{o} & 2.75^{o}

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APPENDIX

For horizontal stratified free channel flow in tubes [4] applies

$$(E - y)(A_L)^2 = Q_L^2/(2g) \qquad [4]$$

since A_L is a function of y the liquid depth.

For example, in the region $0.5 \leqslant y/r \leqslant 1.5$ where A_L is a straight line function of y,

$$A_L = 4.3584\,y - 1.8190 \qquad [30]$$

for a tube diameter of 4.54 cm which corresponds to the main body of data which are reported in this work.

Thus [4] becomes,

$$(E - y)(4.3584\,y - 1.8190)^2 = Q_L^2/(2g) \qquad [31]$$

which is a constant for any given liquid volumetric flow rate, so [31] represents a series of curves which are bounded by and asympotes to the lines,

$$\left.\begin{aligned} E &= Y \\ Y &= 0.1417. \end{aligned}\right\} \qquad [32]$$

As y/r drops below 0.5 note that the latter equality of [32] will be reduced in value as [30] no longer applies.

To find the critical depth of flow, values of Q_L are assumed in [31]. The exact values chosen in this work corresponded to the conditions under which the majority of the holdup data were obtained. The r.h.s. of [31] is now constant and by assigning values to y the corresponding values of E can be obtained. By trial and error calculation the value of E_c at the minimum point can be obtained, and this corresponds to y the initial depth.

Downward inclined stratified flow in tubes is handled in a similar manner except that [28] applies in this particular case

$$(H - k)A_L^2 = \frac{Q_L^2}{2g\cos\alpha} \qquad [28]$$

Again by assigning a volumetric flow rate and angle of inclination for a given pipe diameter, the right hand side becomes a constant. The liquid flow area A_L is a function of y in the normal way. However k is now related to y,

$$k = y/\cos\alpha. \qquad [33]$$

Thus by assigning values to y it is possible to calculate the corresponding values for H. Trial and error calculations thus will result in finding H_c, the minimum energy which corresponds to the initial depth y_c.